

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- find 10, 100, 1000, 10 000 or 100 000 more or less than a given number
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:

- · read, write, order and compare numbers with up to three decimal places
- identify the value of each digit in numbers given to three decimal places
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- · add numbers mentally with increasingly large numbers
- add decimals, including a mix of one-digit whole numbers and tenths, e.g. 1.3 + 0.5; and complements of 1, e.g. 0.83 + 0.17
- add whole numbers with more than four digits, including using the formal written method (columnar addition)
- add decimals to two places, including using the formal written method (columnar addition)
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- · solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including
 understanding the meaning of the equals sign
- solve problems involving number up to three decimal places

ADDITION Continued



Written methods

- · Add whole numbers with more than four digits
- Add decimals with up to two decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar addition

2 957 + 14 635	56.47 + 84.84
12957	56.47
+ 14635	+ 84.76
27592	141.33
1 1	1 1 1

Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', 'carry one thousand' or 'carry ten thousand', not 'carry one'.

Where appropriate, place value columns are labelled, e.g. TO-th, to remind children of the value of each of the digits.

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- subtract numbers mentally with increasingly large numbers
- subtract decimals, including a mix of one-digit whole numbers and tenths, e.g. 1.8 0.7
- subtract whole numbers with more than four digits, including using the formal written method (columnar subtraction)
- subtract decimals to two places, including using the formal written method (columnar subtraction)
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels
 of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including
 understanding the meaning of the equals sign
- · solve problems involving numbers up to three decimal places

Mental strategies

- Continue to use models and images when necessary:
 - trios



- decimals addition and subtraction tables

+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.6	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6
0.7	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11
0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13
0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14
0.05	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15
0.06	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16
0.07	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17
0.08	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18
0.09	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19
0.1	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	0.2

- · Develop further the relationship between addition and subtraction
- Calculate mentally a difference such as 12 462 2300 by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 16 9 = 7 and $1 \cdot 6 0 \cdot 9 = 0 \cdot 7$
- Use partitioning, e.g. 456 84 = 456 80 4

SUBTRACTION Continued

Written methods

- Subtract whole numbers with more than four digits
- Subtract decimals with up to two decimal places
- · Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

45 257 – 17 488	83.72 - 36.49
3 14 11 14 17	7 13 6 12
45257	<u>83 · 15</u>
<u> </u>	<u>- 36 · 49</u>
27769	47.23

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens, then the hundreds, then the thousands and finally the tens of thousands. Refer to subtracting the tens, for example, by saying '14 tens subtract 8 tens', not '14 subtract 8'.

In the first example the ones, tens, hundreds and thousands to be subtracted are all larger than all of the ones, tens, hundreds and thousands you are subtracting from.

The calculation begins by exchanging one of the 5 tens for 10 ones, crossing out the 5 and writing a superscript 4, and crossing out the 7 and writing a superscript 17. The calculation then becomes 17 subtract 8.

You then exchange one of the 2 hundreds for 10 tens, crossing out the 2 and writing a superscript 1, and writing a superscript 1 in front of the 4 to make 14 tens. The calculation then becomes 14 tens subtract 8 tens.

Next, you exchange one of the 5 thousands for 10 hundreds, crossing out the 5 and writing a superscript 4, and writing a superscript 1 in front of the 1 to make 11 hundreds. The calculation then becomes 11 hundreds subtract 4 hundreds. Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 4 to make 14 thousands. The calculation then becomes 14 thousands subtract 4 hundreds are superscript 3.

7 thousands. Then finally 30 000 subtract 10 000.

Where appropriate, place value columns are labelled, e.g. TO-th, to remind children of the value of each of the digits.

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of the multiplication facts for multiplication tables up to 12 × 12
- use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, e.g. 70 × 80 = 5600
- continue to use place value, known and derived facts to multiply mentally
- multiply whole numbers and those involving decimals by 10, 100 and 1000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- · know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- continue to recognise commutativity in mental calculations
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels
 of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems, involving multiplication including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates

MULTIPLICATION Continued

Mental strategies

• Continue to use models and images when necessary:

36 48

- trios

-



24

12

multiplication square to 12 x 12/multiples of 10 multiplication square

60

72

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

10

108 120 132 144

96

84

11 12

- · Continue to use the inverse relationship between multiplication and division
- Use related facts and doubling and halving:
 - double or halve the most significant digit first
 - to multiply by 25, multiply by 100 then divide by 4
 - double one number and halve the other
- find the multiplication facts for the x 16 multiplication table by doubling the x 8 multiplication facts
- Use closely related facts:
 - multiply by 19 or 21 by multiplying by 20 and adjusting
- develop the x 14 multiplication table by adding facts from the x 12 and x 2 multiplication tables
- Use factors, e.g. 9 x 18 = 9 x 6 x 3
- Use patterns of similar calculations, e.g. $8 \times 6 = 48$ and $8 \times 600 = 4800$ •
- Understand and use the commutative law, e.g. 14 x 12 = (2 x 7) x 12

Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one digit . number, or two two-digit numbers, e.g.

 $78 \times 34 = (78 \times 30) + (78 \times 4)$ $378 \times 4 = (300 \times 4) + (70 \times 4) + (8 \times 4)$ = 1200 + 280 + 32 = 2340 + 312 = 1512 = 2652

MULTIPLICATION Continued

Written methods

- Short multiplication:
- Multiply numbers up to four digits by a one-digit number (HTO × O/ThHTO × O)
- · Estimate and check the answer to a calculation

Grid method

Expanded written method

4

32

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 378 × 4 it is 'seventy multiplied by four', not 'seven multiplied by four', although the relationship 7 × 4 should be stressed).

 (8×4)

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

378 × 4

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

• Long multiplication:

- Multiply numbers up to three digits by a two-digit number (TO × TO/HTO × TO)
- · Estimate and check the answer to a calculation

Grid method

78 × 44

					78	
×	70	8		×	34	
30	2100	240	2340 →	23	340	(78 × 30)
4	280	32 +	312 →	3	312	(78 × 4)
			2652 →	26	652	

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 78 is multiplied by 30 (using knowledge of 78 x 3), then 78 is multiplied by 4, and finally the two products are added together.

MULTIPLICATION Continued

Written methods continued

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of the division facts for multiplication tables up to 12 x 12
- use known division facts to derive related facts involving multiples of 10, 100 and 1000, e.g. 6300 ÷ 90 = 70
- · continue to use place value, known and derived facts to divide mentally
- divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- · identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- · know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels • of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates

Mental strategies

Continue to use models and images when necessary:

trios

 $12 \times 7 = 84$ $84 \div 12 = 7$ $84 \div 7 = 12$

multiplication square to 12 x 12/multiples of 10 multiplication square

	×	2	3	4	5	6	7	8	9	10	11	12		×	20	30	40	50	60	70	80	90	100	110	120
	1	2	3	4	5	6	7	8	9	10	11	12		1	20	30	40	50	60	70	80	90	100	110	120
Γ	2	4	6	8	10	12	14	16	18	20	22	24		2	40	60	80	100	120	140	160	180	200	220	240
Γ	3	6	9	12	15	18	21	24	27	30	33	36		3	60	90	120	150	180	210	240	270	300	330	360
Γ	4	8	12	16	20	24	28	32	36	40	44	48		4	80	120	160	200	240	280	320	360	400	440	480
Γ	5	10	15	20	25	30	35	40	45	50	55	60		5	100	150	200	250	300	350	400	450	500	550	600
Γ	6	12	18	24	30	36	42	48	54	60	66	72		6	120	180	240	300	360	420	480	540	600	660	720
	7	14	21	28	35	42	49	56	63	70	77	84		7	140	210	280	350	420	490	560	630	700	770	840
	8	16	24	32	40	48	56	64	72	80	88	96		8	160	240	320	400	480	560	640	720	800	880	960
Γ	9	18	27	36	45	54	63	72	81	90	99	108		9	180	270	360	450	540	630	720	810	900	990	1080
Γ	10	20	30	40	50	60	70	80	90	100	110	120		10	200	300	400	500	600	700	800	900	1000	1100	1200
Γ	11	22	33	44	55	66	77	88	99	110	121	132		11	220	330	440	550	660	770	880	990	1100	1210	1320
Γ	12	24	36	48	60	72	84	96	108	120	132	144		12	240	360	480	600	720	840	960	1080	1200	1320	1440
Con	tinu	ue to	o us	e th	e in	vers	se re	elati	onsl	hip I	oetw	/eer	n mu	ltipl	cati	on a	and	divis	sion						

DIVISION Continued

Mental strategies continued

- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number,
 - $486 \div 9 = (450 \div 9) + (36 \div 9)$
 - = 50 + 4
 - = 54

Written methods

- Short division (including with remainders expressed as a whole number, fraction or decimal):
- Divide numbers up to 4 digits by a one-digit number (HTO ÷ O/ThHTO ÷ O)
- Estimate and check the answer to a calculation

Formal written method of short division

Whole number remainder	Fraction remainder	Decimal remainder
$279 \div 6$	$279 \div 6$	$279 \div 6$
4 6 r 3	4 6 $\frac{1}{2}$	46.5
6) <u>2</u> 7 ³ 9	6) 2 7 39	6) 2 7 ³ 9 . ³ 0

Children should describe what they are doing using phrases similar to the following: 'How many sixes divide into 270 so that the answer is a multiple of 10? (40) There are 40 sixes or 240, with 30 remaining.' The superscript 3 represents the 3 tens that are remaining after 6 has been divided into 270. It is written in front of the 9 to show that 39 now has to be divided by 6.

Children then ask: 'How many sixes in 39?' (6 remainder 3). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.